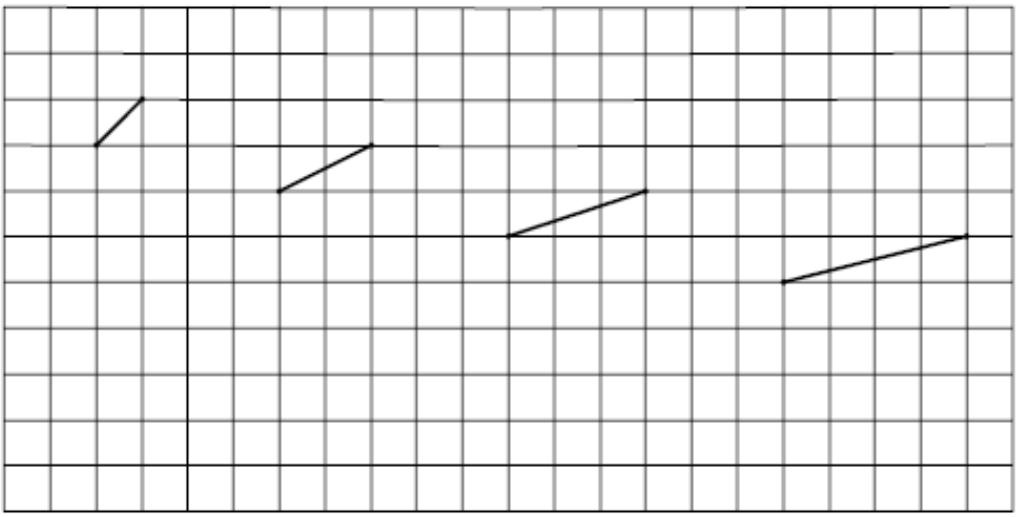
Grade 8 Mathematics Worksheet

Measurement and construction

Questions:

1. For grids A, B and C, complete the following steps for each line segment
   - Let the slope of each segment be $\frac{a}{b}$.
   - Write the slope, in unsimplified form, next to the segment. In Grid A, a = 1; in Grid B, a = 2 ; in Grid C, a = 3.
   - Using your knowledge of parallel and perpendicular lines, build a square that is on the upper left side of each segment.
   - Divide each square into a composite of right triangles and squares by drawing segments in from the vertices, along the horizontal or vertical grid lines.
   - Find the area of the original square from the sum of the areas of the composite figures.
   - Find the length of each original segment from the area of its square.

Grid A Slope = $\frac{1}{b}$



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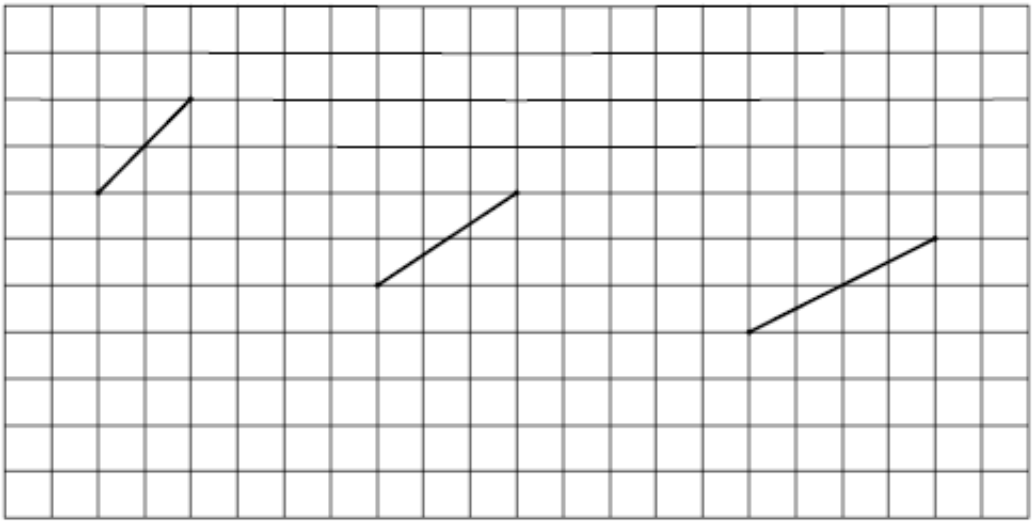
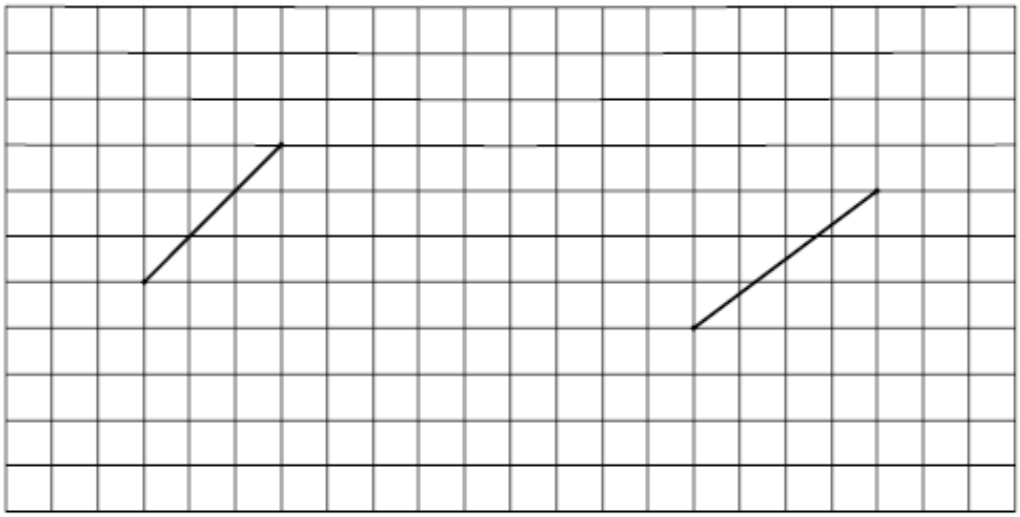
Grid B Slope = $\frac{2}{b}$



Grid C Slope = $\frac{3}{b}$



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2. Complete the following table. Write the length of each of the original segments in unsimplified radical form (i.e. $\sqrt{2}$, rather than 1.14 …) In your group, using the grid figures and the table, discuss and determine relationships among slope numbers, area and segment length.

	Slope	**Area of Original Square**	**Length of Original Segment**
Grid A Slope = $\frac{1}{b}$			
Grid B Slope = $\frac{2}{b}$			
Grid C Slope = $\frac{3}{b}$			

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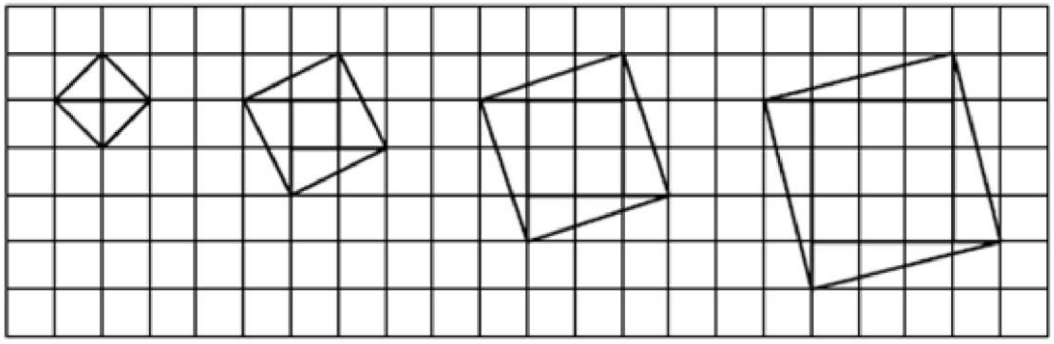
Solution

Grid A Slope = $\frac{1}{b}$



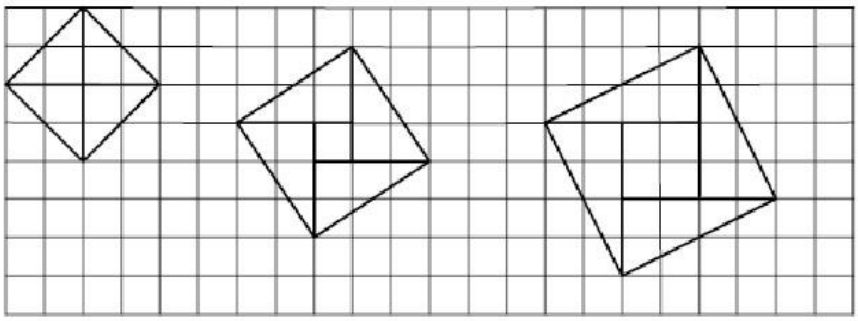
Slope = $\frac{1}{1}$
Area of square
= 4 • $\frac{1}{2}$
= 2
Length = $\sqrt{2}$

Slope = $\frac{1}{2}$
Area of square
= (4 • 1) + 1
= 5
length = $\sqrt{5}$ = 2.24

Slope = $\frac{1}{3}$
Area of square
= $\left(4.\frac{3}{2}\right)$ + 4
= 10
length = $\sqrt{10}$ = 3.16

Slope = $\frac{1}{4}$
Area of square
= (4.2) + 9
= 17
length = $\sqrt{17}$ = 4.12

Grid B Slope = $\frac{2}{b}$



Slope = $\frac{2}{2}$
Area of square
= 4(2)
= 8
Length = $\sqrt{8}$ = 2.83

Slope = $\frac{2}{3}$
Area of square
= 4(3) + 1
= 13
length = $\sqrt{13}$ = 3.61

Slope = $\frac{2}{4}$
Area of square
= 4(4) + 4
= 20
length = $\sqrt{20}$ = 4.47

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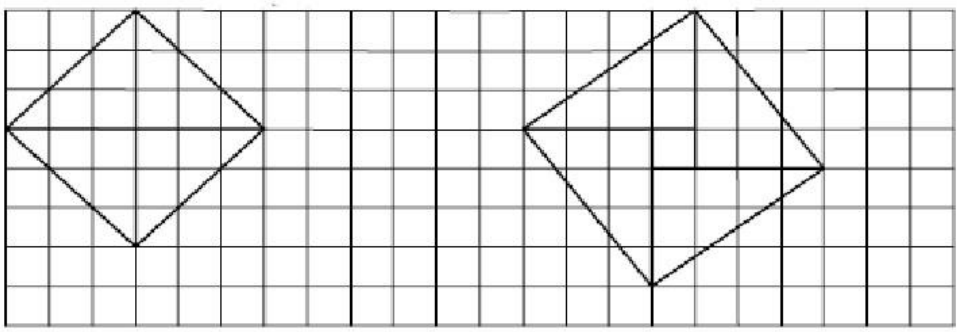
Grid C Slope = $\frac{3}{b}$



Slope = $\frac{3}{3}$

Area of square

= 4 . $\frac{9}{2}$

= 18

Length = $\sqrt{18}$ = 4.24

Slope = $\frac{3}{4}$

Area of square

= (4 . 6) + 1

= 25

length = $\sqrt{25}$ = 5

2.

	Slope	Area of Original Square	Length of Original Segment
Grid A Slope = $\frac{1}{b}$	$\frac{1}{1}$	2	$\sqrt{2}$
	$\frac{1}{2}$	5	$\sqrt{5}$
	$\frac{1}{3}$	10	$\sqrt{10}$
	$\frac{1}{4}$	17	$\sqrt{17}$
Grid B Slope = $\frac{2}{b}$	$\frac{2}{2}$	8	$\sqrt{8}$
	$\frac{2}{3}$	13	$\sqrt{13}$
	$\frac{2}{4}$	20	$\sqrt{20}$
Grid C Slope = $\frac{3}{b}$	$\frac{3}{3}$	18	$\sqrt{18}$
	$\frac{3}{4}$	25	$\sqrt{25=5}$

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The relationships that are evaluated are:

The area of the square is equal to the sum of the squares of the numbers in the unsimplified slope ratio. To generalise, if the segment's slope (in unsimplified form) is $\frac{a}{b}$, then the area of the square on the segment is $a^2 + b^2$.
The length of the segment is given by $\sqrt{a^2 + b^2}$.
Other relationships that may emerge:

In the grid figures, when the slopes are of the form $\frac{a}{a}$, there is no middle square, and the area is given by $2\ a^2$.
When the numbers in the slope ratio differ by 1, the area of the middle square is 1^2.
When the numbers in the slope ratio differ by 2, the area of the middle square is 2^2.
When the numbers in the slope ratio differ by 3, the area of the middle square is 3^2.
The base and the height of each of the four congruent triangles are always the two numbers in the unsimplified slope ratio.

To generalise, if slope = $\frac{a}{b}$ (in unsimplified form), the area of the middle square is $(a - b)^2$; the area of each of the four congruent triangles is $\frac{ab}{2}$.