

Grade 7 Maths Worksheet

Exponential notation and laws

Questions:

1. One of your classmates really struggles with exponents and gave the following answers to problems:

i) $(2)^{-4} = -8$

ii) $3^6 \times 3^{-4} = 9^{-24}$

iii) $\frac{10^{-2}}{10^{-4}} = 1^2$

Explain to your friend **why** the answers are wrong and give the correct answers to each of the problems.

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Solution

1. i) $(2)^{-4} = -8:$

My friend is reading this as a product between the two numbers. So he is saying $(2) \times (-4) = -8$. It is actually a product, but one that is repeated. There are two features that he needs to be aware of. The negative in the power, does not indicate that the number is negative, but that we are working with the reciprocal of the 2. So instead of working with 2, we work with $2^{-1} = \frac{1}{2}$. The 4 in the power

indicates that this $\frac{1}{2}$ must be multiplied by itself 4 times. So that

$$(2)^{-4} = (2^{-1})^4 = \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = \frac{1 \times 1 \times 1 \times 1}{2 \times 2 \times 2 \times 2} = \frac{1}{16}$$

ii) $3^6 \times 3^{-4} = 9^{-24}:$

My friend is multiplying the bases together (by ignoring the powers) to get a new base, and then he is multiplying the powers together (ignoring the bases) to get a new power. There are two possible approaches that I can take to help my friend.

Approach 1:

$3^6 \times 3^{-4} = 3^{6-4} = 3^2 = 9$. This is based on the fact that we are multiplying like bases. The rule indicates that if you multiply like bases, you need to add the powers.

Approach 2:

$3^6 \times 3^{-4} = \frac{3^6}{3^4} = \frac{3 \times 3 \times 3 \times 3 \times 3 \times 3}{3 \times 3 \times 3 \times 3} = \frac{3 \times 3}{1} = 9$. You can make the power of the 3 positive, and break it up into the different powers, and simplify the expression by removing the common factors. Here you will be using the rule $a^{-m} = \frac{1}{a^m}$.

iii) $\frac{10^{-2}}{10^{-4}} = 1^2:$ My friend is under the impression that he can cancel the two 10's as

common numbers. He is thinking that $\frac{10}{10} = 1$ and is ignoring the fact that there are powers over the 10's. And then the powers are subtracted to get $-2 - (-4) = -2 + 4 = 2$

My approach will be to use the rule for the division of like bases which says "if the bases are the same, and we are dividing them, we must subtract the

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powers". So then $\frac{10^{-2}}{10^{-4}} = 10^{-2-(-4)} = 10^{-2+4} = 10^2 = 100$. We can also make the powers positive first and then cancel the common factors or apply the rule. So

$$\frac{10^{-2}}{10^{-4}} = \frac{10^4}{10^2} = \frac{10 \times 10 \times 10 \times 10}{10 \times 10} = \frac{10 \times 10}{1} = 100 \quad \text{or} \quad \text{then}$$

$$\frac{10^{-2}}{10^{-4}} = \frac{10^4}{10^2} = 10^{4-2} = 10^2 = 100.$$

This type of question is extremely difficult for learners to answer as they are expected to evaluate another person's work and also comment on their mistakes, which are sometimes the same mistakes and misconceptions that they have.

It is thus very important that the feedback on this question is done referring to all the possible errors that can be made based on various limiting constructions and misconceptions.

The solution includes some multiple approaches to address this.

Appendix of Assignment Tools

Exponential laws governing the multiplication, division of like bases

Distributive laws regarding the powers

Simplification of quotients